Study of the mathematical pendulum (remote experiment)

Tasks:

a) Determine the value of acceleration due to gravity.

b) Find out how the period of a mathematical pendulum depends on its length.

Remote experiment available at [http://ises4.prf.ujep.cz/.](http://ises4.prf.ujep.cz/)

Measurement principle

By a mathematical pendulum we mean a material point m, which is fixed on an immaterial hinge and moves due to the action of the gravitational force. The formula for the period T of a mathematical pendulum of length l is valid

$$T = 2\pi\sqrt{\frac{l}{g}}, \qquad (1)$$

in which g is the acceleration due to gravity.

The mathematical pendulum can be approximated by suspending a small ball (with a radius of less than 1 cm) from a thin, perfectly strong thread whose mass is negligible relative to the mass of the ball. In this way we can get very close to the conditions under which equation (1) was derived.

A more accurate calculation yields an expression for the period of the oscillation T that contains, in addition to the quantities of the length of the pendulum l and the magnitude of the gravitational acceleration g, the amplitude of the deflection α

$$T = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{4}\sin^2\frac{\alpha}{2} + \ldots\right) \qquad (2)$$

The deviation from equation (1) is more pronounced the larger the amplitude of the deflection α. With friction and resistance to air, the divergence becomes smaller and smaller, but the oscillation period remains the same.

Experiment description

Distant experiment Mathematical pendulum consists of a body suspended on a hinge. The body is a brass sphere of mass (250 ± 1) g and diameter (40.0 ± 0.1) mm. The length of the hinge is adjustable from 800 mm to 1 637 mm. Two initial deflections can be set for the pendulum. The passages of the ball through the equilibrium position are recorded by means of an optical barrier. In the remote experiment, three readings are recorded: time, deflection, and the value '0' or '1'. In the time interval during which the sphere passes through the barrier (the beam is broken), the pendulum deflection is '0'; if the sphere is outside the optical barrier, the deflection is '1'. The period of the pendulum, i.e. the period of the oscillation, can be determined either from the recorded passes of the sphere through the equilibrium position (e.g. the time interval when the deflections have values 0-1-0-1) or from the values of the sensed deflection.

The position of the ball deflection is sensed by a so-called water potentiometer. The water potentiometer consists of a tray filled with plain water. On the edges of the tray, there are lead electrodes. A symmetric voltage of +5 V and -5 V is applied to them. The voltage is evenly "spread" on the surface between the electrodes. The surface of the water is penetrated by a needle that is galvanically attached to the bottom of the swinging sphere. The sphere is suspended from a thin metal conductive thread. The needle, which functions as a potentiometer, moves on the surface of the water and senses a variable potential from about -5 V to + 5 V. In equilibrium, the voltage is 0 V. The detected voltage is measured with a voltmeter.

The record from the water potentiometer is sinusoidal oscillations, the record from the optical barrier is pulses. Both signals are sensed by the ISES measurement system. The remote experiment not only allows visual monitoring of the experiment, but also enables data recording and export to Excel, where the data can be further processed as needed.

Only one user at a time can perform measurements on a remote experiment. The first user to connect to the remote experiment can use it individually for 20 minutes. If it shows no activity for a longer period of time (mouse movement, data capture, data transfer, etc.), it is automatically disconnected. If another user wants to join the experiment, he is in the so-called queue, and after the first user's 20 min time has elapsed, the next user is automatically connected to the remote experiment. The remaining time of the first user is displayed on the control WWW page.

Measurement procedure

1. Set the length of the pendulum (e.g., 80 cm).
2. Use the PREPARE 1 or PREPARE 2 button to set the size of the initial horizontal deflection of the pendulum. Select the deflection setting according to the pendulum length setting so that the initial angular deflection does not exceed 4° - verify this by calculation.
3. Press the START RECORD button to start data recording.
4. Use the RELEASE button to start the experiment.
5. Use the STOP RECORD button to stop data recording.
6. Export the measured data to an Excel file using the VALUES IN CVS USING COMAS button.
7. Repeat the measurement for five different pendulum lengths (ranging from 80 to 160 cm).
8. For each pendulum length, determine the period (period of oscillation) of the pendulum. You can find it, for example, by finding the second maximum value in the recorded values (2nd column of the excel spreadsheet), which corresponds to the maximum deflection to the opposite side from which the pendulum was initially launched (this will be the first positive value). Then find the next positive maximum value between which a time interval equal to one period has elapsed. Find the next positive maximum value in a similar way. To reduce the error, consider several consecutive periods (e.g.

five) and determine the time interval corresponding to them. Then divide this by the number of periods to get the time of one period.

9. For each length of the pendulum, calculate the acceleration of gravity, determine its average value, and the statistical error of measurement (probable error). Compare the value of the acceleration of gravity thus determined with the tabulated value of the acceleration of gravity in Ústí nad Labem.
10. Find out how the period of the oscillation depends on the length of the pendulum. Plot the dependence of the period of kmit on the square root of the length of the pendulum.
11. Discuss the results obtained with respect to possible systematic errors in the measurement method used.
12. Attach the exported Excel files to the report.
13. Recommendation: Try the experiment at least twice "for real" before recording the values you will process.

Instructions for Processing Measurements

Compare the observed value of the acceleration of gravity g with the value found in the tables (it is best to find directly the value of the acceleration of gravity in a latitude place that has a similar to the place where the measurement is made) or compare it with the value calculated using the Hayford relation

$$g = 9{,}780\,49\,(1 + 0{,}005\,288\,4\,\sin^2\alpha - 0{,}000\,005\,9\,\sin^2 2\alpha) - 0{,}000\,001\,967\,h,$$

where α is the latitude and h is the altitude of the place where the acceleration is measured.

Similar remote pendulum experiments are located all over the world. You can find them at the link https://elab.vps.tecnico.ulisboa.pt:8000/apparatuses.

Try using them to find out what the acceleration of gravity is in e.g. Brazil or Delhi.