Remote physics experiment Mathematical pendulum as an attractive alternative to traditional laboratory exercises

F Lustig[1]**, P Brom**[2] **and E Hejnová**[1]

[1] Department of Physics, Faculty of Science, Jan Evangelista Purkyně University in Ústí nad Labem, Czech Republic
[2] Department of Quantitative Methods, ŠKODA AUTO University, Czech Republic

Abstract. Traditional laboratory exercises are usually not popular among students, especially the processing and submission of protocols. Students are used to using digital technologies, prefer active participation in the learning process, and love visualisations. To improve motivation and make laboratory practices more attractive to students, remote laboratories with the real time control and available anytime and anywhere can be offered. In this paper, we present the remote experiment Mathematical Pendulum, which is physically located at the Department of Physics, Faculty of Science, Jan Evangelista Purkyně University (UJEP) in Ústí nad Labem. Using it we can study physical laws such as the dependence of the pendulum's period of oscillation on its length and determine the value of the gravitational acceleration. The remote pendulum is interactively controlled through a web browser from a computer or mobile phone, provides live video stream and experimental data downloadable via Internet. This single remote laboratory task is complemented by remote measurements of the acceleration of gravity at different latitudes through a group of ten identical mathematical pendulums created within the WorldPendulum WP@ELAB project. In this paper, we present not only the results of the measurements made, but also a pilot investigation of the use of this complex remote experiment.

1. Introduction – history of pendulums

The history of a pendulum dates back to antiquity. The first mention of pendulums appears in ancient Egypt, regarding their use in the construction of temple doors. In China, pendulums were also used to maintain direction during travel.

Galileo Galilei was considered to be the first scientist to study pendulums experimentally. Around 1582, he observed the pendulum motion of a chandelier in the cathedral of Pisa. He measured time using the beats of his heart because there was no accurate clock. He found that the period was independent of the mass of the pendulum as well as its amplitude; however, it was dependent on the length of the suspension.

The first pendulum clock was constructed in 1656 by the Dutch scientist Christiaan Huygens. The best accuracy achieved was around 15 seconds per day. In 1671, the French astronomer Jean Richer, during his expedition from Paris to Cayenne, French Guiana, found that the pendulum clock was 2.5 minutes per day slower on Cayenne than in Paris. He concluded that the gravitational force was lower on Cayenne. (Note: study of this relationship in a school is also the goal of the WP@ELAB project described below.)

In the mid-19th century, the pendulum was also used to measure gravitational acceleration. Pendulums are still used in various fields such as precision clocks, gyroscopes, seismographs and other devices. Due to the dependence of the period on temperature, air pressure, and the effect of air friction on the motion of the pendulum as well, pendulum clocks have been replaced since 1927 by inexpensive and affordable quartz crystal oscillator clocks. Today, accurate timekeeping is done with atomic clocks with extremely high stability and accuracy (deviation 1 ns per day).

2. Pendulum as a remotely controlled experiment

Remote laboratories became part of laboratory classes many years ago. The first remote laboratories were described in [1]. Probably the most interesting contribution to remote pendulum realisations was the German World Pendulum project [2]. An interesting idea here was the deployment of remote pendulums at different latitudes. These pendulums were placed in Germany (Kaisersesch, Hermannsburg), Italy (Naples), Yemen (Aden), and Latvia (Riga). The pendulums consisted of a steel sphere, a wire and a precise suspension. Their length of 2.7 m corresponded to a pendulum period of one second. The initiation of the oscillation was carried out by a moving electromagnet on a sledge moving on rails. When the electromagnet was switched off, the sphere was released to oscillate. An important part of the experimental setup was an optical gate for the measurement of the pendulum period in order to determine the Earth's gravitational acceleration g. The accuracy of the time measurement of about 0.1 ms was determined using the sampling rate of counters connected to the optical gate. Gravitational acceleration was measured at different latitudes. These experiments were created in 2007 and are no longer available now.

Another interesting remote pendulum was in Slovakia (in Trnava) [3]. The angular displacement readings were made using two dynamometers. The period of the pendulum's oscillation was measured with an optical barrier. The measurements with dynamometers also enabled the acquisition of the sinusoidal oscillation waveform. The task was to determine the local gravitational acceleration, to study the dependence of the period of the oscillation on the initial deflection angle, to determine the kinetic, potential and total mechanical energy, and to study the damping caused by air friction. Unfortunately, this remote laboratory is no longer online.

In 2019, a unique project called WP@ELAB, the World Pendulum Alliance (WPA) [4] was introduced, a federated initiative of 20 technical universities to create a network of pendulums at different latitudes to map local gravitational differences. (Here is the full list of locations and technical universities where remote measurements can be performed: Barcelona – UPC, Bogotá – UNAD, Bogotá – UniAnde, *Brasilia – UnB*, Ciudad de Panamá – USMA, Ciudad de Panamá – UTP, Díli – EPT, Faro – CCVAlg, Ilheus – UESC, *Lisbon – Planetário*, Maputo – EPM, *Marseille – ECM*, Oeiras – IST, Prague – CTU, Praia – UniCV, *Punta Arenas – UMag*, Rio de Janeiro – PUC2, Rio de Janeiro – PUC, Santiago – UChile, *São Tomé – EPSTP*, *Valparaiso – USM*; italic denotes sites where the authors performed remote measurement – see further). An interactive map of the locations is available at [5]. Not all of them are available at the moment. The sites are chosen to cover as wide a range of latitudes as possible. Most of them are in South America.

In [6] there is a list with remote pendulums that includes all necessary data (geographical location, altitude), and all parameters of pendulum (length of the suspension, mass and diameter of the sphere, reference temperature, thermal expansion coefficient, temperature in the laboratory, etc.). Any user can have free access to any remote measurements on all WPA pendulums (no password, but a Google account is required).

The goal of remote measurement is, of course, to measure gravitational acceleration as a function of geographic location. More precise tasks such as temperature dependence of the suspension length, damping by air friction, tidal dependence on the position of the Moon and Earth, etc. can also be studied.

3. Our pendulum as a remote experiment

Probably one of the most sophisticated remote pendulum experiments is located in our laboratory at the Faculty of Science at the Jan Evangelista Purkyně University (UJEP) in Ústí nad Labem. The experiment has been launched recently (2021), is fully functional and can be accessed 24/7 using a standard browser at [7].

3.1. Experimental design

A mathematical pendulum (see figure 1) with a variable suspension length (0.8 m to 1.6 m) and an adjustable initial deflection of 9.5 cm or 6.5 cm. The period of the pendulum is measured using an optical gate, and the deflection is measured using a water-based potentiometer. Under the sphere there is a water reservoir with two lead electrodes, connected to a constant voltage. A miniature needle is attached to the bottom of the pendulum sphere and touches the water surface. The voltage between the position of the needle and one electrode is measured. This voltage is proportional to the deflection of the mathematical pendulum. The experimental setup is monitored by two webcameras that provide a live video stream.

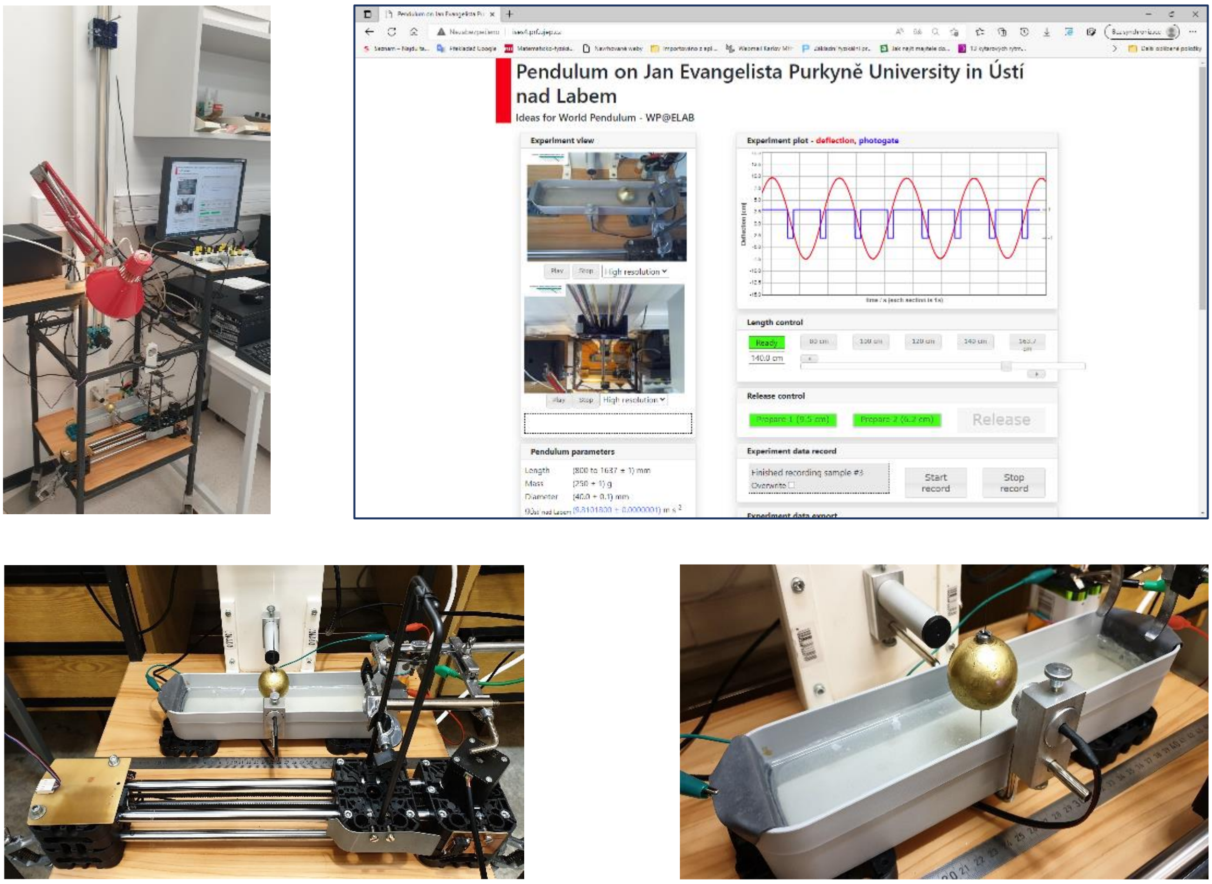

Figure 1. Experimental setup of the remote pendulum at the Faculty of Science, UJEP in Ústí nad Labem (Czech Republic, 2021). On the left is a view of the whole experimental setup; the adjustable pendulum length mechanism can be seen. The bottom left is the pendulum sphere with an optical gate and a motorized initial deflection mechanism, and the bottom right is a detailed view of the water-based potentiometer. The upper right is the control web interface (client screen) of the remote laboratory experiment. The remote experiment can be accessed from any web browser on the URL address [7] or from the remote experiment signpost [8].

3.2. Software design

The software part of the remote experiment uses the software development kit "iSES Remote Lab SDK", [9]. The server part of the remote experiment consists of the MeasureServer and ImageServer applications. MeasureServer provides bidirectional communication with the hardware, ImageServer provides live video stream. The real-time data connection is implemented using WebSocket technology, and a web server (e.g., the free Nginx) is also installed. Clients connect to the remote experiment using common web browsers. The web interface (client screen) of the remote lab experiment is also shown in figure 1.

The software of the remote experiment is in JavaScript, so that the remote experiment can be run not only on computers but also on mobile phones.

The practical tasks of this remote experiment are to determine the local gravitational acceleration, to measure the dependence of the period of the oscillation on the length of the suspension, on the initial deflection angle, and to determine the kinetic, potential, and total mechanical energy of the pendulum, or also to study the damping caused by air friction and on the water potentiometer.

4. Theory

According to the literature, the experimental value of the Earth's gravity varies between 9.78 $\mathrm{m{\cdot}s^{-2}}$ at the Equator and approx. 9.83 $\mathrm{m{\cdot}s^{-2}}$ at the Poles. Many various and less or more complicated formulas can be found for the theoretical computation of the Earth's gravity, e.g., Carlo Somigliana's formula, or Gino Casinnis's International Gravity Formula for Hayford ellipsoid (used in USA since 1930) [8], [10]

$$g_{\left[\mathrm{m\cdot s^{-2}}\right]}(\varphi,h)=9.780\,49\cdot\left[1+0.0052884\cdot\sin^2\varphi-5.9\cdot10^{-6}\cdot\sin^2(2\varphi)\right]-1.967\cdot10^{-6}\cdot h_{[\mathrm{m}]} \tag{1}$$

where φ is the latitude and h denotes the height above the sea level (in other words the altitude, elevation). Other examples of series expansions can be found in [10], including the dependence on the mean rock density. Let's mention the latitude and height dependence by the representative organization of national legal metrology authorities for Europe called WELMEC, based on the International Gravity Formula (IUGG, 1967, Luzern) [10]:

$$g_{\left[\mathrm{m\cdot s^{-2}}\right]}(\varphi,h)=9.780\,318\cdot\left[1+0.0053024\cdot\sin^2\varphi-5.8\cdot10^{-6}\cdot\sin^2(2\varphi)\right]-3.085\cdot10^{-6}\cdot h_{[\mathrm{m}]} \tag{2}$$

In addition, the formulas (1–2) mentioned above illustrate typical variability of the parameters in series expansions. For educational purposes, these variations can be neglected. The value of gravity measured at the terrestrial surface is the result of a combination of the following factors: 1. The gravitational attraction of Earth as a whole, 2. Centrifugal force caused by Earth's rotation, 3. Elevation, 4. Unbalanced attractions caused by surface topography, 5. Tidal variations and 6. Unbalanced attractions caused by irregularities in underground density distributions. [11]

Physics and description of the motion of a mathematical pendulum belong to basic course of physics. Usually from the solution to the differential equation with the standard approximation $\sin\alpha\approx\alpha$ where α denotes the angular deflection in radians, always satisfying the condition $\alpha(t) < 0.07$ rad, $\alpha(t) < 4°$, we can learn the dependence among the time period T, the length L of the mathematical pendulum, and the local gravitational acceleration g

$$T=2\pi\sqrt{\frac{L}{g}} \tag{3}$$

including the fact that the period T does not depend on the weight of the pendulum. From the experimental determination of T and L, it is possible to compute the local gravitational acceleration

$$g=\frac{4\pi^2\cdot L}{T^2}. \tag{4}$$

Furthermore, we need to consider the other relevant conditions that may affect some of these physical quantities. The length of the string l is the function of the temperature t_{lab} in the laboratory, which can be described with increasing linear function

$$l = l(t_{\text{lab}}) = l_0 \cdot \left[1 + \alpha_{\text{string}} \cdot \left(t_{\text{lab}} - t_0\right)\right] \tag{5}$$

where α_{string} is the thermal expansion coefficient of the string material, and l_0 denotes the reference length of the string at a certain reference temperature t_0 in the laboratory. The total length L of a mathematical pendulum is the distance between the axis of revolutions (oscillations) of the pendulum and the centre of gravity of a small heavy sphere representing the pendulum. From the simple geometry (see the figure 2c) it holds

$$L = l + \frac{D}{2} \tag{6}$$

where D denotes the diameter of the sphere and l is the length of the light string whose weight can be neglected to the weight of the sphere. The values of the parameters in equations (1–6) can be found in [6].

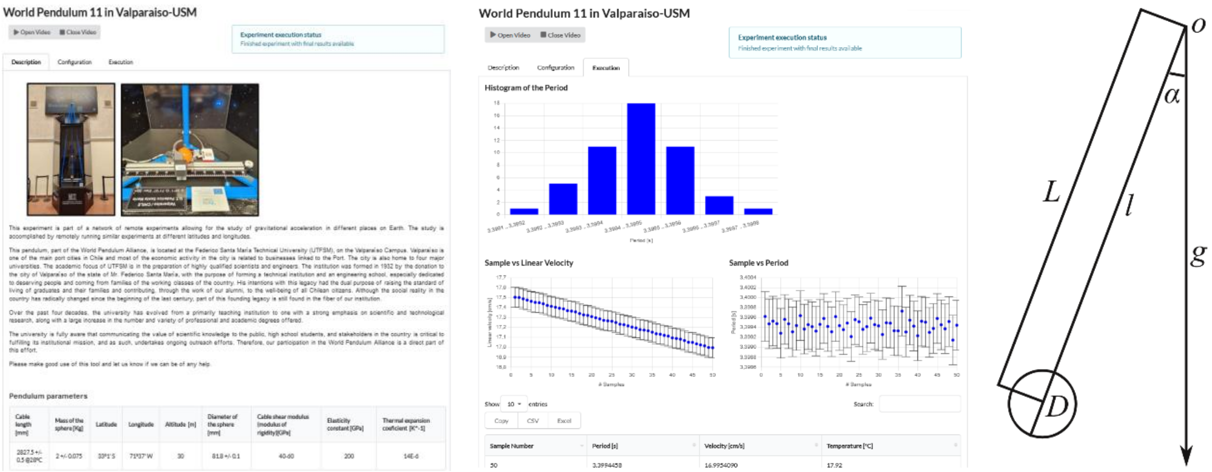

Figure 2. (a) left: web page of a WP-project pendulum with all the necessary parameters (a screenshot), (b) mid: overview of experimental data (N = 50) including a histogram of the period T usually with the symmetry, decrease in velocity, and thus the deflection amplitude due to air friction, and constant value of the period versus sample index, (c) right: descriptive physical quantities for a pendulum.

5. Measurement of gravity using a mathematical pendulum

5.1. World pendulum (WP@ELAB)

The data processing procedure using equations (2–6) was considered in advance in order to repeat the entire procedure with all experimental data sets that always have the same format [6]. Prior to the processing, the quality of experimental data was checked using more graphical overviews (see examples of graphs and histograms in the figure 2b). The experimental data were downloaded in the Excel format, so the necessary formulas and computations were implemented on the same Excel sheet. Subsequently, the sheet with all necessary formulas could be copied, and experimental data and parameters could be simply overwritten with data from another world pendulum. For each of these pendulums, the experimental value of period T could be determined using histograms and averages: both values corresponded in general. Typical results can be seen and compared with the theoretical values according to the equation (2) in the graph in the figure 3. Unfortunately, there is a lack of information and insufficient space for a detailed discussion of the uncertainities and possible sources of errors, which is challenging. Surprisingly, the data from EPSTP (São Tomé, Gulf of Guinea) were affected with significantly greater variability than the data from the other world pendulums, but similar to the uncertainty determined by the excellent students at UJEP (see the graph and the subsections below). Hopefully, some world pendulums and their e-texts will be improved in the future.

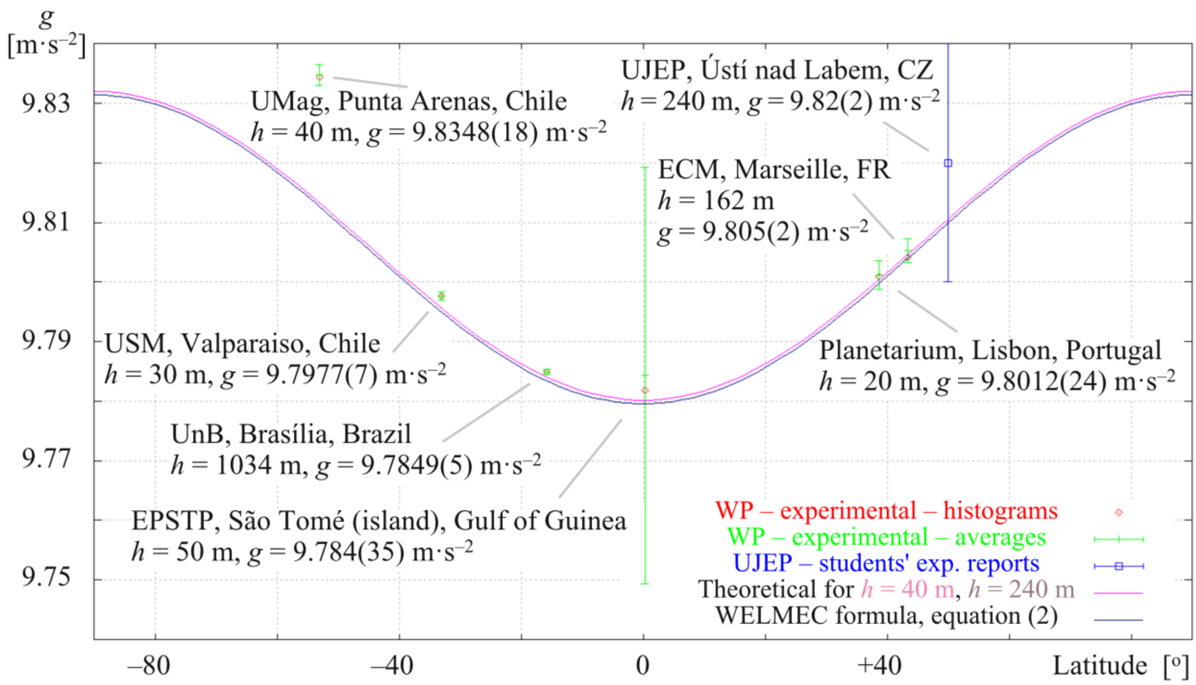

Figure 3. Comparison of the theoretical and typical experimental values for selected remote pendulums.

5.2. Measurement at UJEP, Czech Republic

Remote experiment the mathematical pendulum is included at the Faculty of Science of the J. E. Purkyně University in the Practical course in mechanics and thermics, which is intended for students of the full-time and combined bachelor's degree programme Physics for Education. Students usually take it during the summer term of their first year. The remote experiment is the first time most students encounter this practice during their studies so far. Students have the choice between performing a local measurement with a mathematical pendulum located in the laboratory and a measurement using a remote experiment, which they can perform either directly during the course session or independently from a location of their choice (e.g., from home). Our experience so far shows that the majority of full-time students choose the second option, that is, to make measurements using a remote pendulum, most often choosing to work on their laptop under the supervision of the lecturer as part of a regular class of the course. On the other hand, combined students, who are mostly unqualified middle-aged teachers teaching in primary schools, choose more often the option of local measurements.

The aim of this experimental problem is to determine the value of the local gravitational acceleration and to find out how the period of the mathematical pendulum depends on its length. From the remote experiment, students get a large number of values that are recorded in an Excel spreadsheet. They have to process a large data set, which is usually a new experience for them. Their task is to find an adequate methodology to search for the values of the pendulum deflections that correspond to one period of the pendulum in the set of measured values. To help them do this, the graphs from the remote experiment (see figure 1) allow students to analyse and interpret the results of the experiment. Processing of the measurement also includes a graphical representation of the dependence of the oscillation period of the mathematical pendulum on the square root of its length.

As part of the measurement processing, students must also determine the error (uncertainty) of the result and discuss it with respect to possible error sources in the measurement method used. The method error usually reflects the inaccuracy in determining the pendulum length and the inaccuracy of determining the pendulum period. Because students repeat the measurements for each pendulum length at least five times, they can also determine the statistical error and the total error. Finally, they can compare the observed value of gravitational acceleration with the theoretical value, which they can look up on the Internet or calculate using equation (1). The experiment allows students to compare the measured values with theoretical predictions to better understand the physical laws associated with the mathematical pendulum.

5.3. Discussion

The didactic benefits of remote experiment measurement include, in particular, its flexibility and accessibility, that is, the possibility to carry out measurements without limitations of time and place, which can be particularly helpful for students who for various reasons cannot attend face-to-face classes. This allows students to choose their own individual pace when measuring, and last but not least, this approach supports their independent work and lesson planning. An undoubted benefit of the remote experiment is the possibility of its visualization, which provides students with a better understanding of the behaviour of the pendulum and the physical phenomena associated with it. In addition, the possibility of student interaction with the pendulum (students can change its length and the size of the initial deflection) increases the attractiveness of remote measurement and the motivation of students to carefully process the measurement data.

However, it is important to mention some of the limitations that the remote experiment entails. These lie primarily in the lack of physical contact with the actual pendulum. The remote experiment usually provides students with the same experience as an experiment with a real pendulum because it loses the opportunity to directly observe and measure physical quantities with real measuring instruments. This may also be the reason why older students, who are usually no longer part of the generation that has been online from an early age, prefer real experiments. Furthermore, when discussing possible errors in remote measurements, systematic errors commonly found in real experiments cannot be discussed, which can limit the students' understanding of the practical aspects of experimental measurement. This fact may to some extent eliminate the demonstration of the remote pendulum to the students so that they have the opportunity to become more familiar with the remote experiment and actually observe the experiment in progress. In the protocol, they can then discuss possible factors that affect the measurement results such as air friction, needle friction on the water surface, non-linear trajectory of the sphere, etc. Therefore, when using a remote experiment, it is important to always consider the benefits and limitations compared to a traditional experiment, allowing its effective and innovative integration into physical practice.

Students who worked in a physics practicum with a remote mathematical pendulum in the 2022/23 academic year also provided valuable feedback to us. From the reactions of full-time students, it is obvious that they find this form of measurement an interesting refresher in an otherwise traditionally conceived practical course, which for them is often a somewhat monotonous sequence of measurements of usually traditional tasks, followed by generally time-consuming processing of results. In contrast, a certain distance and distrust towards this form of experiment was evident among the students of combined studies.

In order to implement remote experiments in a laboratory course, it is necessary to reliably ensure that the experiment is easily accessible online for students and that it is maintained in a working state. Therefore, the experiment should be tested and maintained regularly to avoid technical obstacles. Students should also be provided with clear instructions on the problem, explaining the basic physics principles and the measurement procedure so that they can work independently. [8]

Experiences with the use of remote experiment in university physics practice have given us interesting insights into the benefits but also the limitations of this modern didactic method. The flexibility, accessibility, interactivity of the remote experiment, and the possibility of experimental verification of theoretical knowledge are key aspects that significantly enrich the measurement in practical course and increase its attractiveness. Although there are some limitations associated with the lack of physical contact with the actual experiment, remote experimentation is an effective and innovative tool in physics practice that seeks to use modern technology for the benefit of students who can learn the skills of scientific work well.

6. Conclusion

In this paper we have presensed various remote experiments to study the dependence of local gravitational acceleration on latitude (World Pendulum project, WP@ELAB) and on the length of suspension, and on the deflection of pendulum and other laws of physics (UJEP, Czech Republic). The main feature and advantage of the World Pendulum project is the uniform hardware and user interface, which allow simple automatization of data processing by worldwide users, making remote measurements and data processing very effective. The mathematical pendulum with the goals of determining the experimental value of the local gravitational acceleration and verifying relevant physical dependences is a common part of practical courses at many schools. We recommend all these schools to combine both local and remotely controlled measurements with mathematical pendulums, as was suggested in this contribution.

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